Cosmological singularities and modified theories of gravity

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Abstract. We consider perturbative modifications of the Friedmann equations in terms of energy density corresponding to modified theories of gravity proposed as an alternative route to comply with the observed accelerated expansion of the universe. Assuming that the present matter content of the universe is a pressureless fluid, the possible singularities that may arise as the final state of the universe are surveyed. It is shown that, at most, two coefficients of the perturbative expansion of the Friedman equations are relevant for the analysis. Some examples of application of the perturbative scheme are included.

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INTRODUCTION

Recent astronomical data from Type Ia supernovae [1] as well as from the CMB spectrum [2] confirm that our universe is undergoing an accelerated expansion period. In order to comply with this feature, one may resort to postulating a dark energy content for the universe [3], with undesired properties, such as violation of some energy conditions, or going beyond general relativity in the quest for another theory of gravity [4, 5, 6, 7].

With either approaches for dealing with the observed accelerated expansion, cosmology is much richer than it was thought in the previous century. According to classical cosmologies, the universe started at an initial singularity, the Big Bang, and it was doomed to expand forever, since the matter content was not dense enough to stop expansion and collapse into a final singularity.

However, accelerated expansion of the universe has lead to consider other plans for the end of the universe in the form of Big Rip singularities [8], directional singularities [9] or milder sudden singularities [10]. It is therefore interesting to tell under which circumstances a modified theory of gravity may lead to such fate for the universe.

MODIFIED GRAVITY

Instead of dealing with a full theory of modified gravity, we focus on the consequences at the level of cosmological equations, namely Friedmann equation, just requiring that it

admits a generalised power expansion on the density ρ around a value ρ_*,

$$\left(\frac{\dot a}{a}\right)^2 = H^2 = h_0(\rho-\rho_*)^{\xi_0} + h_1(\rho-\rho_*)^{\xi_1} + \cdots, \quad \xi_0 < \xi_1 < \cdots. \tag{1}$$

The standard density term arises as the lineal term with an exponent equal to one and the cosmological constant appears with a null exponent in this expansion. Further terms are interpreted as modifications of the theory.

On the other hand, the energy conservation law implies

$$\dot\rho + 3H(\rho + p) = 0, \tag{2}$$

but assuming that the accelerated expansion of the universe is solely due to the modification of the theory of gravity, we choose a pressureless dust as matter content of the universe, so that the scale factor of the universe and the density are related by $\rho a^3 = K$.

Thereby we may get rid of the scale factor and write down a modified Friedmann equation in terms of density:

$$\frac{\dot\rho}{\rho} \;=\; -3\sqrt{h_0}(\rho-\rho_*)^{\xi_0/2} - \frac{3}{2}\frac{h_1}{\sqrt{h_0}}(\rho-\rho_*)^{\xi_1-\xi_0/2} + \cdots \quad . \tag{3}$$

Solving this equation provides a perturbative expansion of the density in coordinate time, which we want to compare with a similar expansion for the scale factor:

$$a(t) = c_0|t-t_0|^{\eta_0} + c_1|t-t_1|^{\eta_1} + \cdots, \quad \eta_0 < \eta_1 < \cdots,$$

in terms of coordinate time. This is useful, since in [11] we have related the exponents η_i with the strength of singularities according to the standard definitions by Tipler [12] and Królak [13], as we see in Table 1. The strength of the singularities just points out if tidal forces are strong enough to disrupt finite objects on crossing them.

TABLE 1. Singularities in cosmological models

η_0	η_1	η_2	**Tipler**	**Królak**	**N.O.T.**
$(-\infty,0)$	(η_0,∞)	(η_1,∞)	Strong	Strong	I
0	$(0,1)$	(η_1,∞)	Weak	Strong	III
	1	$(1,2)$	Weak	Weak	II
		$[2,\infty)$	Weak	Weak	IV
	$(1,2)$	(η_1,∞)	Weak	Weak	II
	$[2,\infty)$	(η_1,∞)	Weak	Weak	IV
$(0,\infty)$	(η_0,∞)	(η_1,∞)	Strong	Strong	Crunch

The last column refers to the classification of future singularities in [14]):

- Type I: "Big Rip": divergent a.
- Type II: "Sudden": finite a, H, divergent $\dot H$.
- Type III: "Big Freeze": finite a, divergent H.
- Type IV: "Big Brake": finite a, H, $\dot H$, but divergent higher derivatives.

We have chosen the decreasing density branch of the Friedmann equation, since we wish to mimic the actual expansion phase of the universe. There are two possibilities: either there is a critical value ρ_* for which the density stops decreasing or it goes on decreasing without a lower bound. The results are consigned in tables 2 and 3. More details may be found in [15].

Singularities in models with null critical density are determined by the sign of the first exponent ξ_0. If it is positive, expansion goes on forever and no singularity arises, but if the exponent is negative, a Big Rip singularity comes up due to the accelerated expansion.

TABLE 2. Singularities in models with $\rho_* = 0$

ξ_0	**Tipler**	**Królak**	**N.O.T.**
$(-\infty, 0)$	Strong	Strong	I
$[0, \infty)$	Non-singular	Non-singular	Non-singular

On the contrary, the structure of models with a finite critical density ρ_* is much richer. No Big Rip singularities appear and they are all weak. Therefore, they cannot be considered the final stage of the universe [16].

Models with negative ξ_0 exponent have divergent H, which is a Big Freeze singularity, which is weak under Tipler's definition, but strong with Królak's.

Models with null ξ_0 exhibit a cosmological constant term and produce just weak singularities. The second exponent ξ_1 can be used to tell the derivative of H which is singular, but none of these can be taken as the final fate of the universe, due to the weakness of the singularity.

Finally, models with no cosmological constant, $\xi_0 > 0$ show the same types of weak singularities.

Examples of proposed models belonging to these families may be found in [15].

TABLE 3. Singularities in models with $\rho_* \neq 0$

ξ_0	ξ_1	**Tipler**	**Królak**	**N.O.T.**
$(-\infty, 0)$	(ξ_0, ∞)	Weak	Strong	III
0	$(0, 1)$	Weak	Weak	II
	$[1, \infty)$	Weak	Weak	IV
$(0, 1)$	(ξ_0, ∞)	Weak	Weak	II
$[1, 2)$	(ξ_0, ∞)	Weak	Weak	IV

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